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TEMPERATURE AND MASS OF POLYTROPIC STARS IN GRAVITATION EQUILIBRIUM

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Abstract The Relation between the central temperature and the central density is investigated for a polytropic stars and a polytropic (or isothermal)stellar core, which are in gravitation equilibrium . In this paper we have demonstrated that the mass of the stars keeps on increasing from its surface to centre. Approximate analytic solutions to the equilibrium equations have been presented in phase planes such as (U_m,V_T), Transformations connecting solutions in this phase plane have been obtained and discussed.

Index Terms Polytropes stars , stellar core, Phase Plane, Transformations connection, equilibrium equations.

1. Introduction

THHROUGHOUT the life of a stars, the central temperature and density change to a considerable extent. The The features of the overall evolution of the stars are determined mainly by how far the central temperature rises in its whole life and accordingly how far the synthesis of the chemical element proceeds in the interior. The behavior of solutions of the Lane-Emden equations is polytropic index n, which controls the distribution of physical variables, has been studied by Hopf [1], Fowler [2] and Chandrasekhar for n<3, n=3, and n>3, respectively. It is well known so far from some of these studies that the polytropic index n=0 and 1 represent, the liquid and gaseous states of a polytrope of uniform density respectively. The origin and the behavior of Lane-Emden equations were reported same whatever be the index of a polytrope[3-14]. The Miline [14] was able to determine the maximum limiting density [15] and the maximum value of mass of a star[16] for n$\rightarrow$ 0 and $\rightarrow$1 whereas the structure of planet was also reported[17,18] for the same values of n. Further thermo dynamical equilibrium of stars clusters embedded in an isothermal configuration[19], relativistic stellar structures and X-ray transients in Ni's theory of gravity [20], very massive stellar models in Ni's theory of gravity[21], and general relativity neutron star [22] were also reported for the same values of n.

The theory of polytropes in which the pressure (P) and density (ρ) are related by a monomial relation of the kind, $P = K\rho^{1+\frac{1}{n}}$ (n and K are two disposable constants; n is the polytropic index, and K defines the temperature implicitly) may be considered as a fundamental parameters to the study of stellar structures.

Considering the stars, which are in equilibrium and in a steady state can be characterized by three physical parameters i.e. its mass M; its radius R; and its luminosity L(L refers to the amount of radiant energy in ergs , radiated by the star per second to the space outside,) analytic series solutions to the equilibrium equations have been presented in phase planes such as (U_m,V_T), Transformations connecting solutions in this phase plane have been obtained, Since the nucleus includes the immediate neighborhood of the origin (n=0), it will be of the interest to investigate it, in the light of this new concept of uniform density for n -> 0 and n -> 1.

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2. Structure Equation in (U_m,V_T)phase plane

The equations governing the structure of a polytopic configuration of index n with angular velocity Ω can be expressed with the help of electromagnetic Maxwall's equations

$$P = K\rho^{1+\frac{1}{n}} \quad (1)$$

$$\nabla^2 \varphi = -4\pi G\rho \quad (2)$$

$$\frac{P}{\rho} = \nabla \vartheta + \frac{1}{2}\Omega^2 X^2, X^2 = x^2 + y^2 \quad (3)$$

where, P is the pressure, ρ the density, ϕ the gravitational potential, X the distance from the axis of rotation, K a constant, and G the gravitational constant (6.67 x 10^{-8} dynes cm^2/gm^2). If we introduce ϒ as the distance from the centre of the polytrope, and define the dimensionless variable θ, and ξ_θ by the relations

$$\rho = \rho_c \theta^n ; \gamma = \alpha \ \xi = \left[\frac{(n+1)\,k}{4\pi G} \rho_c^{\frac{1}{n}-1} \right]^{1/2} \xi \qquad (4)$$

$$\omega = \frac{\Omega^2}{2\pi G \rho_c}$$

where ρ_c is the central density.

From equations (1) (2) (3)&(4) we can deduce the following expression in (V_T ,U_m) Phase Plane.

$$\frac{1}{V_T^N} \frac{d}{dV_T} \left(V_T U_m{}^N \frac{dU_m}{dV_T} \right) = -U_m{}^n + \omega \qquad (5)$$

Which satisfied the boundary conditions

$$U_m = 1 , \ \frac{dU_m}{dV_T} = 0 \ \text{at } V_T = 0 \qquad (6)$$

Equation (5) is known as general "Lane-Emden" Equation for polytropic index n. For the convenience we take, Also equation (5) can be written as,

$$\frac{N}{V_T} \frac{dU_m}{dV_T} + \frac{d^2 U_m}{dV_T{}^2} = -U_m{}^n + \omega \qquad (7)$$

For non-rotating case, $\omega = 0$ as $\Omega = 0$

Equation (7) becomes

$$\frac{N}{V_T} \frac{dU_m}{dV_T} + \frac{d^2 U_m}{dV_T{}^2} = -U_m{}^n \qquad (8)$$

Equation (8) is the required structure equation, for non-rotating case, in (U_m, V_T) phase plane.

Here we solve the structure Equation for polytropic index n = 1 and N=2 (spheroidal), (N = 1) (cylindrical) and for N = 0, (plane symmetric)

Case-1 for n = 1 and N = 2, equation (8) becomes,

$$\frac{2}{V_T} \frac{dU_m}{dV_T} + \frac{d^2 U_m}{dV_T{}^2} = -U_m \qquad (9)$$

applying the boundary conditions

$$\text{for } U_m = 1 , \qquad \frac{dU_m}{dV_T} = 0 \ \text{at } V_T = 0 \qquad (10)$$

The series solution of the form, satisfying the boundary conditions can be expressed as

$U_m = 1 + a_1 V_T^2 + a_2 V_T^4 + a_3 V_T^6 + a_4 V_T^8 + \ldots\ldots\ldots\ldots$

Putting the Value of a_1, a_2, a_3, a_4, a_5 and a_6 in equation (11) we get, the required solution.

$$U_m = 1 + a_1 V_T^2 + a_2 V_T^4 + a_3 V_T^6 + a_4 V_T^8 + a_5 V_T^{10} + a_6 V_T^{12} + \ldots\ldots\ldots \qquad (11)$$

Differentiating equation (11) w. r. t. V_T we get

$$\frac{dU_m}{dV_T} = 2a_2 V_T + 4a_2 V_T^3 + 6a_3 V_T^5 + 8a_4 V_T^7 + 10a_5 V_T^9 + \ldots\ldots \qquad (12)$$

again differentiating equation (12) w.r.t V_T we get,

$$\frac{d^2 U_m}{dV_T^2} = 2a_1 + 12a_2 V_T^2 + 30a_3 V_T^4 + 56a_4 V_T^6 + 90a_5 V_T^8 + 132\, a_6 V_T^{10} + \ldots\ldots\ldots \qquad (13)$$

From equations (9), (11), (12) & (13) we get

$$\frac{2}{V_T}\left(2a_1 V_T + 4a_2 V_T^3 + 6a_3 V_T^5 + 8a_4 V_T^7 + 10a_5 V_T^9 + 12a_6 V_T^{11} + \ldots..\right)$$
$$+\left(2a_1 + 129a_2 V_T^2 + 30a_3 V_T^4 + 56a_4 V_T^6 + 90a_5 V_T^8 + 132a_6 V_T^{10} + \ldots..\right)$$
$$= -(1 + a_1 V_T^2 + a_2 V_T^4 + a_3 V_T^6 + a_4 V_T^8 + a_5 V_T^{10} + a_6 V_T^{12} + \ldots\ldots)$$

$6a_1 = -1 \quad \Rightarrow \quad a_1 = -\frac{1}{6}$

$20a_2 = a_1 \quad \Rightarrow \quad a_2 = \frac{1}{120}$

$43a_3 = -a_2 \quad \Rightarrow \quad a_3 = -\frac{1}{5040}$

$74a_4 = -a_3 \quad \Rightarrow \quad a_4 = -\frac{1}{362880}$

$110a_5 = -a_4 \quad \Rightarrow \quad a_5 = -\frac{1}{39916800}$

$156a_6 = -a_5 \quad \Rightarrow \quad a_6 = -\frac{1}{6227020800}$

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Equating the co-efficient of powers of V_T, we get,

$$U_m = 1 - \frac{1}{6}V_T^2 + \frac{1}{120}V_T^4 - \frac{1}{5040}V_T^6 + \frac{1}{362880}V_T^8 \text{...........(14)}$$

Case-2 for cylindrical shape i.e. N = 1 and n = 1, equation (8) becomes

$$\frac{1}{V_T}\frac{dU_m}{dV_T} + \frac{d^2U_m}{dV_T^2} = -U_m \qquad (15)$$

series solution can be expressed as

$U_m = 1 + a_1 V_T^2 + a_2 V_T^4 + a_3 V_T^6 + a_4 V_T^8 + a_5 V_T^{10} + a_6 V_T^{12} + \ldots\ldots$
(16)

Differentiating above w. r .t. V_T

$$\frac{dU_m}{dV_T} = 2a_1 V_T + 4a_2 V_T^3 + 6a_3 V_T^5 + 8a_4 V_T^7 + 10a_5 V_T^9 + \text{.......(17)}$$

putting these values in equation (15)

$$\frac{1}{V_T}\left(2a_1 V_T + 4a_2 V_T^3 + 6a_3 V_T^5 + 8a_4 V_T^7 + 10a_5 V_T^9 + \ldots\ldots\ldots\right)$$
$$+\left(2a_1 + 12a_2 V_T^2 + 30a_3 V_T^4 + 56a_4 V_T^6 + 90a_5 V_T^8 + \ldots\ldots\ldots\right)$$
$$= -\left(1 + a_1 V_T^2 + a_2 V_T^4 + a_3 V_T^6 + a_4 V_T^8 + a_5 V_T^{10} + \ldots\ldots\ldots\right)$$

equating the co-efficient of powers of V_T.

$4a_1 = -1 \quad \Rightarrow \quad a_1 = -\frac{1}{4}$

$16a_2 = -a_1 \quad \Rightarrow \quad a_2 = -\frac{1}{64}$

$36a_3 = -a_2 \quad \Rightarrow \quad a_3 = -\frac{1}{2304}$

$100a_5 = -a_4 \quad \Rightarrow \quad a_5 = -\frac{1}{147{,}45600}$

substituting the value of constants a_1, a_2, a_3, a_4 and a_5 in equation (16) we get

$$U_m = 1 - \frac{1}{4}V_T^2 + \frac{1}{64}V_T^4 - \frac{1}{2304}V_T^6 + \frac{1}{147,456}V_T^8 - \frac{1}{147,45600}V_T^{10} + \text{.....(19)}$$

Case-3 for N = 0 & n = 1, equation (8) becomes,

$$\frac{d^2U_m}{dV_T^2} = -U_m \quad \text{........} \qquad (20)$$

equation (20) is in the form of well known, simple harmonic (SHM) motion equation. Solution of above equation (20) is given by

$$U_m = a_1 \sin V_T + a_2 \cos V_T$$

$$\frac{dU_m}{dV_T} = a_1 \cos V_T - a_2 \sin V_T$$

putting the boundary conditions in above equation i.e. for

$U_m = 1$, $\frac{dU_m}{dV_T} = 0$ at $V_T = 0$, we get

$a_1 = 0$ and $a_2 = 1$ (21)

3. Results and Discussion

graphical representation of (V_T , U_m) plane for N =2 & n=1 (Fig. a) , for N =1 & n=1 (Fig.b) and N=0 & n=1 (Fig. c), where V_T show Temperature and U_m show mass of polytropes. The graphs plotted by our series solution method are in good agreement by the graph with the stellar model. It is evident from the figure that the as Temperature of the polytropes decreases, its mass increases in all the three cases implying that the mass of the stars keeps on increasing as we move from surface to centre. The graph for N=0 , N=1, and N=2 between Um TV has been plotted and found to be in good agreement with the results graph of N=0 (plane Symmetric) N=1 (Cylindrical) N=2(spheroidal) the shape stellar structure [4] of given value.

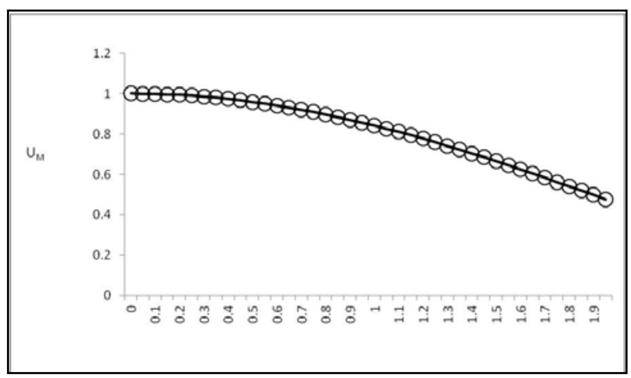

Fig a. Graphical representation of (V_T , U_m) phase plane for N =2 & n=1 where V_T show Temperature and U_m show mass of polytropes, from the figure that the Temperature of the polytropes decreases, its mass increases, mass is more and more centrally condensed, the mass of the polytropes never fall below zero.

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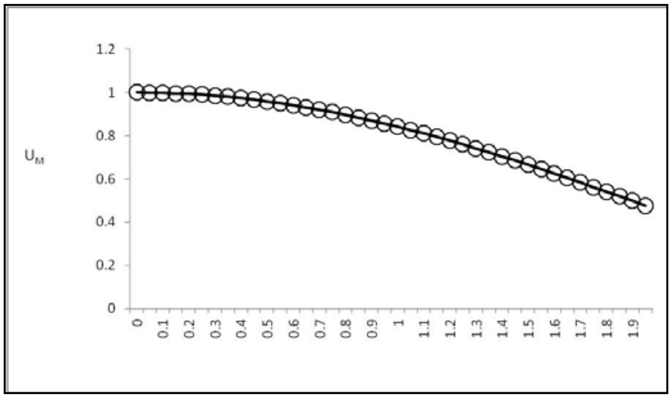

Fig b. Graphical representation of (V_T , U_m) phase plane for N =1 & n=1 where V_T show Temperature and U_m show mass of polytropes, from the figure that the Temperature of the polytropes decreases, its mass increases, mass is more and more centrally condensed, the mass of the polytropes never fall below zero

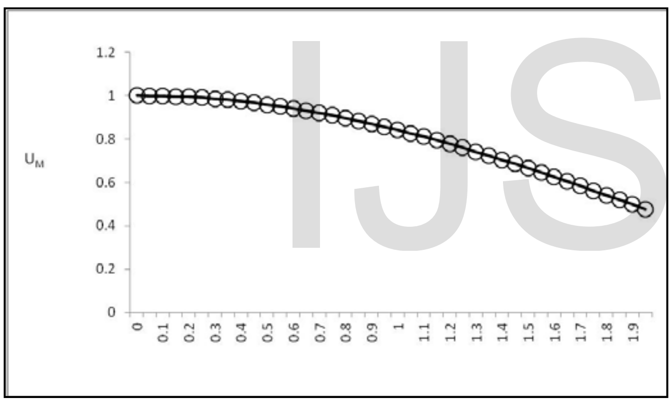

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Fig c. Graphical representation of (V_T , U_m) phase plane for N =0 & n=1 where V_T show Temperature and U_m show mass of polytropes, from the figure that the Temperature of the polytropes decreases, its mass increases, mass is more and more centrally condensed, the mass of the polytropes never fall below zero.

4 Conclusion

An unified analytic study structure of the nucleons of Polytropes N=0 (Plane Symmetric) N=1 (Cylindrical) N=2 (spheroidal) has been investigated following the concept of sphere of uniform density defined by polytropic index (n) tending to zero. The graphs plotted by our series solution method are in good agreement by the graph with the stellar model[4]. The mass of the stars keeps on increasing as we move from surface to centre. Our given analysis can be applied to the interdisciplinary modeling, environmental and biological systems which may quite often involve complicated forms of linear or non-linear differential equation.

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